Joyas Perturbativas en Teoría Cuántica de Campos

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Talk at UNACH

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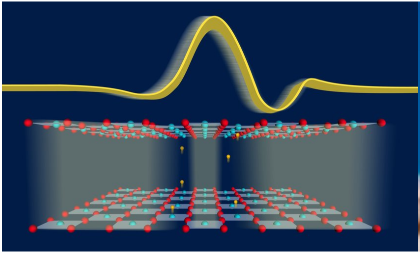
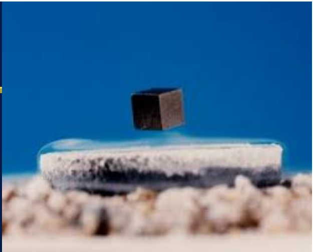
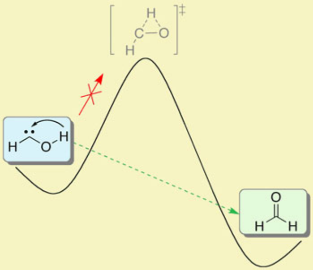
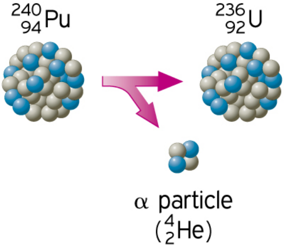
Q is for Quantum ...

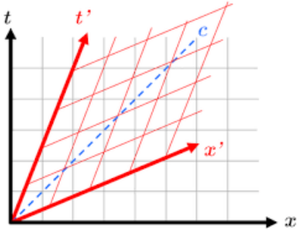


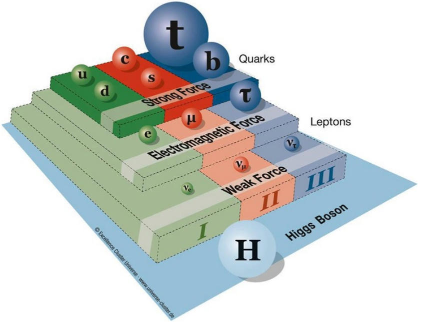


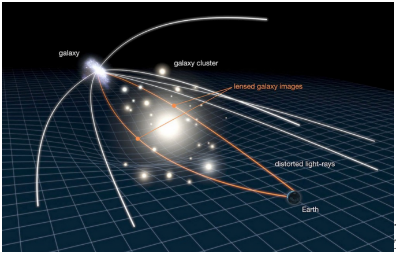


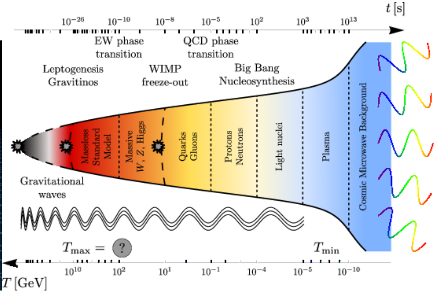


Quantum and Relativity → QFT

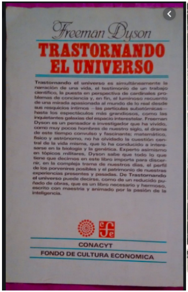


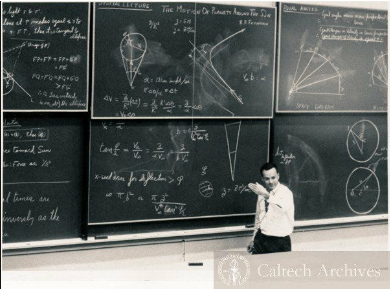






How it started for me? Do I dare disturb the universe?





My main goal: just to understand QFT ...

Outline

- Motivation and QFT roots,
- From Feynman graphs to amplitudes,
- Modern Amplitudes and Perturbative Jewels,
- QFT, KLT and gravitons,
- Conclusions.

From Quantum Mechanics to QFT

- After great success of QM for atoms, molecules, solids, nucleus, Quantum fields were next ..
- Seeds of QFT in Black-body spectrum - Derivation of Einstein coefficients,
- Dirac proposed a relativistic equation for the electron, and: $H = H_e + H_{int} + H_A$,
- First complete formulation of QFT - Heisenberg, Pauli, Jordan

Foundations of QFT

From its very begining QFT has struggled to make sense ... But so far, QFT has survived, with great success (SM).

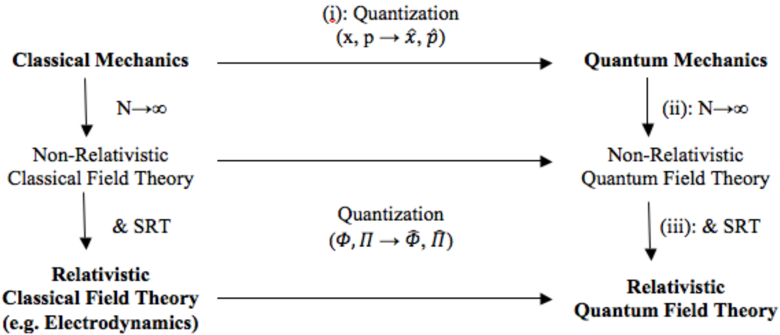
- Is it a theory of particles or fields?
- How to handle the Infinities contained in loops?
- Is Gauge invariance for real? i.e. A_μ vs $A^\pm$,
- Why only a few representations?
- Does the vacuum gravitates?

Generations of theorist have improved QFT calculational methods, extended its applications and deepened our understanding.


QFT Construction - as in many traditional textbooks.

- 1st - Classical Field Theory:
  - Fields are Irreps of Lorentz group,
- 2nd - Cuantization:
  - Canonical: defines the theory, involves operators, CCR $[\phi, \phi^\dagger]$
  - Path Integrals: sum over histories, $< in|out >= \int D\phi e^{iS}$,
- 3rd - Solution:
  - Perturbative (Feynman diagrams)
  - Lattice, instantons, solitons, etc.

QFT - Canonical Quantization



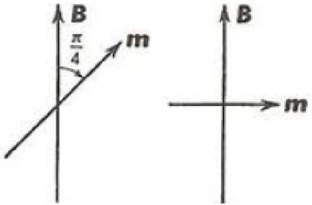
But is this how nature works? or is it just formal? Are fields just an artifact? Are particles real?

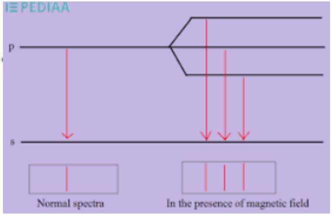
Perturbative solution in Quantum Physics

So, we use a perturbative solutions. What are they?

- Many situations in physics, we can not solve the full problem,
- But luckily, sometimes the Hamiltonian can be splited as: $H = H_0 + H_{int}$.
- Furthermore, H_0 can be solved exactly, e.g. Hydrogen atom,
- Then, H_{int} contains a small parameter and can be taken into account perturbatively, e.g. Hydrogen atom in a magnetic field (Zeeman/Stark effect),

$$\Delta E =< \psi | H_{int} | \psi >= \int d^3x \, \Psi^*(r) H_{int} \Psi(r)$$





Perturbative methods in QFT

- Use pert. language to identify particles, solve: $\mathcal{L}_0$. Ex.

$$\mathcal{L}_0 = \partial\phi\partial\phi - m^2\phi^2 \rightarrow (\partial^2 + m^2)\phi = 0,$$

- Free-particle solution:

$$\phi(x) = \int d^4\tilde{k}[a^\dagger(k)e^{-ik.x} + a(k)e^{ik.x}]$$

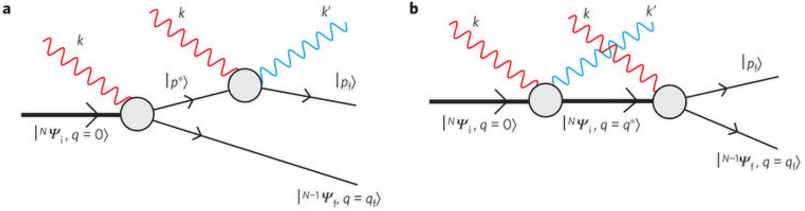
- Canonical quantization: $[a(p), a^\dagger(k)] = C\delta^4(p-k)$, ,
- Hilbert space: $a^\dagger(p)|0 >= |p >$, etc.
- Include interactions with pert. methods, ex. $\mathcal{L}_{int} = \frac{\lambda}{4!}\phi^4$,
- S-Matrix (LSZ) $\rightarrow$ Feynman rules $\rightarrow$ Physical Process,
- No. of particles is not conserved in general, e.g. $e^+e^- \rightarrow q\bar{q} + \gamma$,

Perturbative methods in QFT

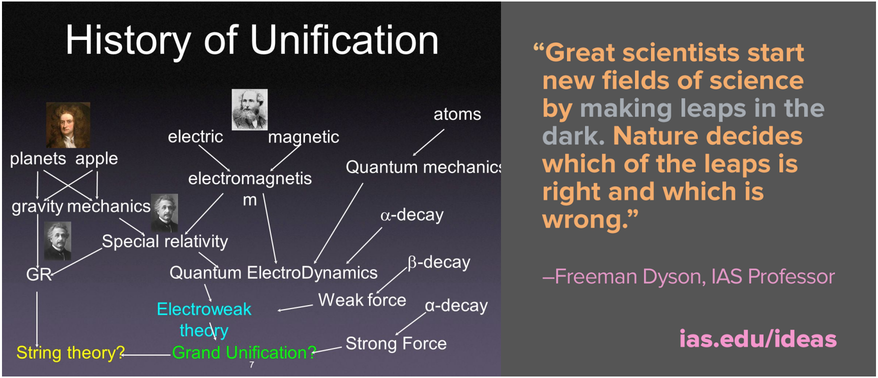
- Amplitude $= \sum$ (Feynman Diagrams)
- Diagram $=$ Ext. Lines $+$ Int. Lines (Propagators) $+$ Vertices,

  Ex. In $\lambda\phi^2$ - (Propagator): $\frac{i}{p^2 - m^2 + i\epsilon}$, Vertex: $-i\lambda$,

- Calculate amplitude, square it, integrate over phase-space $\rightarrow$ Physics observables (cross-sections, lifetimes)



Which QFT for nature?



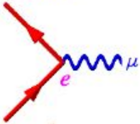
QFT - Quantum Electrodynamics (QED)

Lagrangians in QFT

A catalog of particle fields and their interactions

- Quantum Elecrodynamics: electromagnetism in QFT

$$L = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \tfrac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - e\,\bar{\psi}\gamma^\mu \psi A_\mu$$

Electron "propagator"	Photon "propagator"	Interaction "vertex"
		
$\bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$	$-\tfrac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2$	$-e\,\bar{\psi}\gamma^\mu \psi A_\mu$

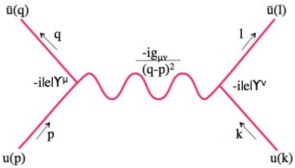
$$\psi(x) = \sum_\sigma \int_{-\infty}^{+\infty} \frac{d^3k}{\sqrt{(2\pi)^3}} \sqrt{\frac{mc^2}{\hbar\omega_k}} \big(\underbrace{c_\sigma(k)}_{\text{particle}} u_\sigma(k) e^{ikx} + \underbrace{d^\dagger_\sigma(k)}_{\text{antiparticle}} v_\sigma(k) e^{-ikx}\big)$$

e- e+ ?

$$c_\sigma(k)\, c_{\sigma'}(k') = -\, c_{\sigma'}(k')\, c_\sigma(k)$$ ← Pauli exclusion principle ?

Amplitudes and Feynman graphs in QED

- Tree-level: $e^+e^- \to \mu^+\mu^-$



- Amplitude:

$$\mathcal{M} = \bar{v}(p_2)(-ie\gamma^\mu)u(p_1)\frac{-ig_{\mu\nu}}{q^2+i\epsilon}\bar{u}(p_3)(-ie\gamma\nu)v(p_4) \qquad (1)$$

- Do the "talacha" (Diracology) and get: $\sigma = \frac{8\pi}{3}\frac{\alpha^2}{s}$, $s = (p_1+p_2)^2$,
- Drawing Feynman graphs is like a "lego" game,

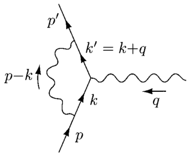
  (Ex. do it for $e^+e^- \to \gamma\gamma$)

Amplitudes and Feynman graphs in QED

- Loop diagrams contain infinities $\rightarrow$ Renormalization,
  (Renormalizables, super-renorm., no-renorm., a few finite theories)
- Ex. Tadpole diagram in $\lambda\phi^4$:

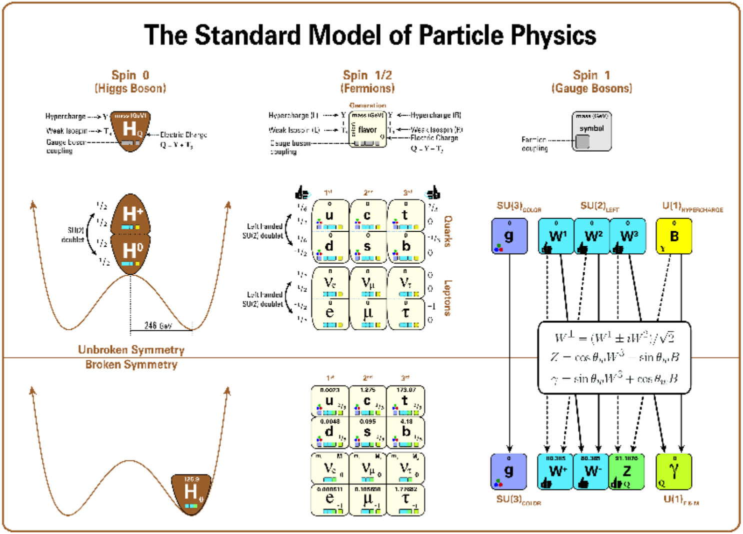
$$\Sigma(p) = \lambda \int d^4q \, \frac{1}{q^2 - m^2} \tag{2}$$

- Finite part of loop diagrams in QED $\rightarrow$ anomalous magnetic moment of electron,



- QED (and QFT) has been proved with a high precision, e.g.
  $a_\mu^{th(exp.)} = (1\,159\,652\,157 \pm 28) \times 10^{-12} [(1\,159\,652\,188 \pm 4) \times 10^{-12}]$,

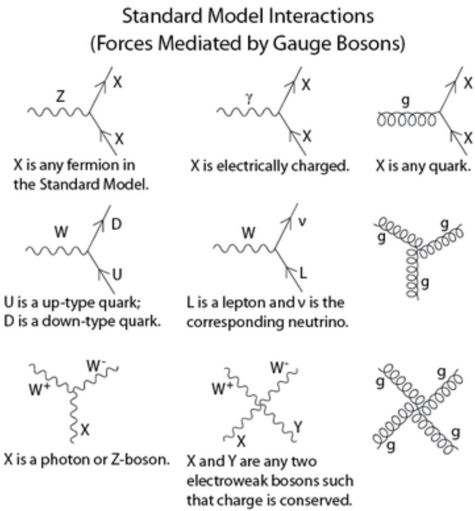
The Standard Model: great success of QFT



Higgs discovery: condensed matter physics in vacuo

The first time that the entire NYT Science section is devoted to a single story

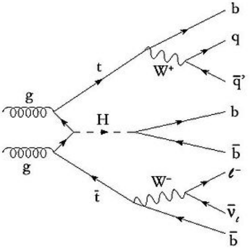
SM Feynman Rules

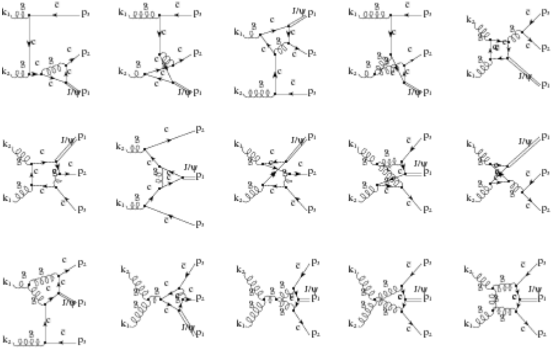


Traditional methods for Pert. QFT

- Draw all diagrams,
- Write down the amplitude(s)
- Square it, sum and average over helicities,
- Integrate over phase space to get cross-sections/Decay widths

"Shut up and calculate" (R Feynman)





Limitations of traditional methods

Examples of processes you have likely encountered in QFT are

$$\begin{aligned} \text{Compton scattering} \quad & e^- + \gamma \;\to\; e^- + \gamma\,, \\ \text{Møller scattering} \quad & e^- + e^- \;\to\; e^- + e^-\,, \\ \text{Bhabha scattering} \quad & e^- + e^+ \;\to\; e^- + e^+\,, \end{aligned} \qquad (1.1$$

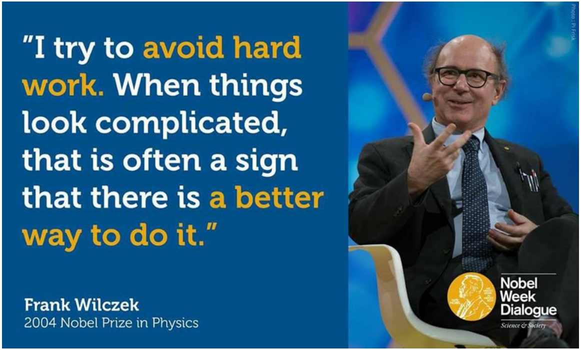
and perhaps also $2 \to 2$ gluon scattering

$$g + g \;\to\; g + g\,. \qquad (1.2$$

But in QCD number of diagrams grows very fast:

$$\begin{array}{lr} g+g \;\to\; g+g & 4 \text{ diagrams} \\ g+g \;\to\; g+g+g & 25 \text{ diagrams} \\ g+g \;\to\; g+g+g+g & 220 \text{ diagrams} \end{array} \qquad (1.3)$$

Is there a better way?



Indeed, in the last decades we have seen the use of modern helicity methods to get lots of practical results and amazing theoretical insights into perturbative QFT (YM, gravity, strings).

Helicity methods - massless case ($p^2 = 0$)

- 4-momentum and Weyl spinors:

$$p_\mu \to p_\mu \sigma^\mu_{a\dot{a}} = p_{a\dot{a}} = |p]_a < p|_{\dot{a}}$$

$$[pk] \equiv [p|^a |k]_a \equiv \phi^a \kappa_a, \;\; \langle pk \rangle \equiv \langle p|_{\dot{a}} |k\rangle^{\dot{a}} \equiv \phi_{\dot{a}} \kappa^{\dot{a}}$$

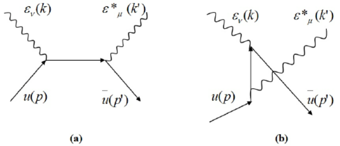
- 4-component spinors:

$$u_-(p) = v_+(p) = \left(\begin{array}{c} |p]_a \\ 0 \end{array} \right), \qquad u_+(p) = v_-(p) = \left(\begin{array}{c} 0 \\ |p\rangle^{\dot{a}} \end{array} \right),$$

- Spinor relations: $\quad \langle q\, p\rangle [p\, q] = 2p \cdot q = (p+q)^2$,
  and $< 1|\gamma^\mu|2 >< 3|\gamma_\mu|4 >= 2 \; < 13 >< 24 >$,
- Photon Polarization vector:

$$\bar{\epsilon}^\mu_+(k) = -\frac{\langle q|\gamma^\mu\,|k]}{\sqrt{2}\langle q\, k\rangle}, \;\; \bar{\epsilon}^\mu_-(k) = -\frac{[q|\,\gamma^\mu\,|k\rangle}{\sqrt{2}[q\, k]}.$$

Compton effect - massless case



- Total Amplitude: $\mathcal{M}_{\lambda_1\lambda_2\lambda_3\lambda_4}$ (with: $1234 \to e_i e_f \gamma_i \gamma_f$)

$$\mathcal{M}_{+-\lambda_3\lambda_4} = (-i)e^2\langle p_2|\varepsilon^\mu_{\lambda_4}(k_4;q_4)(i\gamma_\mu)\left(\frac{-i(\not{p}_1+\not{k}_3)}{(p_1+k_3)^2}\right)(i\gamma_\nu)\varepsilon^\nu_{\lambda_3}(k_3;q_3)\,|p_1]$$
$$-(-i)e^2\langle p_2|\varepsilon^\mu_{\lambda_3}(k_3;q_3)(i\gamma_\mu)\left(\frac{-i(\not{p}_1+\not{k}_4)}{(p_1+k_4)^2}\right)(i\gamma_\nu)\varepsilon^\nu_{\lambda_3}(k_4;q_4)\,|p_1]$$

- Can show: $\lambda_2 = -\lambda_1$, and $\mathcal{M}_{-+\lambda_3\lambda_4}$ can be obtained from $\mathcal{M}_{+-\lambda_3\lambda_4}$ by complex conj.
- Final result:

$$\langle|\mathcal{M}|^2\rangle = 2e^4\left(\left|\frac{s_{14}}{s_{13}}\right| + \left|\frac{s_{13}}{s_{14}}\right|\right)$$
$$= -2e^4\left(\frac{u}{s}+\frac{s}{u}\right).$$

Girl: I like adventurous men.

Me: [Trying to impress her]

ONCE I CALCULATED COMPTON SCATTERING CROSS SECTION WITHOUT IGNORING ANY MASS

MHV amplitudes and Parke-Taylor

Amplitudes for maximal-helicity violation is very simple

The result for the 4-gluon amplitude is an example of the famous ***Parke-Taylor** n**-gluon tree amplitude***: for the case where gluons i and j have helicity -1 and all the $n-2$ other gluons have helicity $+1$, the tree amplitude is

$$A_n[1^+ \dots i^- \dots j^- \dots n^+] = \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}\,. \tag{2.80}$$

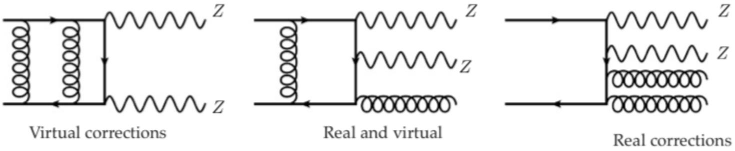
We prove this formula in Section 3. The number of Feynman diagrams that generically contribute to an n-gluon tree amplitude is[9]

$n =$	3	4	5	6	7	…
#diagrams =	1	3	10	38	154	…

(Later on, the formulae was proved with string theory- QFT limit)

Other Jewels based on helicity/amplitude methods -

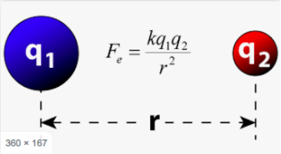
- QCD calculations at LO, NLO, NNLO for LHC, e.g.

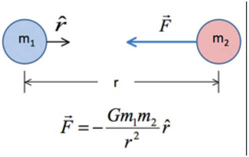


- Unitarity cuts: tree-level $\rightarrow$ loop-level
- What is the simplest QFT? ($N = 4$ SYM)
- BCFW, Twistors, Amplituhedron

Unexpected relations between YM and Quantum Gravity

- KLT Relations (tree-level)
  i.e. GR= (YM)x(YM), Sugra= (SYM)x(SYM),
- Double copy (loop-level)
- Are they really un-expected?





"Gluons almost for nothing and gravitons for free" (JJC)

- Pert, Q. Gravity: $h_{\mu\nu}$ = Graviton,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

- Alternative: start from massless spin-2 state $\rightarrow$ GR (linear),

  (QG as an effective field theory, good enough in the IR)

- Helicity states-
  Gluon/photon: $\epsilon^{\pm}(k)$, Graviton: $\epsilon^{\pm\pm}(k) = \epsilon^{\pm}(k)\epsilon^{\pm}(k)$,

- Graviton propagator:

$$P_{\mu\nu\rho\rho} = \frac{i}{q^2 + i\epsilon}[\eta_{\mu\nu}\eta_{\rho\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\rho}\eta_{\nu\sigma}]$$

[Why $\simeq$ $Tr(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma)$????]

QFT, Gravitons and KLT

- Parke-Taylor for graviton scattering:

$$M_4^{\text{tree}}(1^-2^-3^+4^+) = \frac{\langle 12\rangle^7[12]}{\langle 13\rangle\langle 14\rangle\langle 23\rangle\langle 24\rangle\langle 34\rangle^2} = \frac{\langle 12\rangle^4[34]^4}{stu}.$$

- KLT relations: $hhhh \simeq (gggg)x(gggg)$

$$\begin{aligned} M_4^{\text{tree}}(1234) &= -s_{12}\, A_4^{\text{tree}}[1234]\, A_4^{\text{tree}}[1243]\,, \\ M_5^{\text{tree}}(12345) &= s_{23}s_{45}\, A_5^{\text{tree}}[12345]\, A_5^{\text{tree}}[13254] + (3 \leftrightarrow 4)\,, \\ M_6^{\text{tree}}(123456) &= -s_{12}s_{45} A_6^{\text{tree}}[123456]\Big(s_{35} A_6^{\text{tree}}[153462] + (s_{34}+s_{35}) A_6^{\text{tree}}[154362]\Big) \\ &\quad + \mathcal{P}(2,3,4)\,. \end{aligned}$$

Off-shell vertex: Gauge vs Gravity

3-point gluon vertex

$$\frac{\delta S^3}{\delta A^a_\mu \delta A^b_\sigma \delta A^c_\rho} \to i f^{abc}\left((k_1^\rho - k_2^\rho)\,\eta^{\mu\sigma} + (k_2^\mu - k_3^\mu)\,\eta^{\sigma\rho} + (k_3^\sigma - k_1^\sigma)\,\eta^{\rho\mu} \right) \tag{19}$$

3-point graviton vertex

$$
\begin{aligned}
&\frac{\delta S^3}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma\tau}\delta\varphi_{\rho\lambda}} \to 2\eta^{\mu\tau}\eta^{\nu\sigma}k_1^\lambda k_1^\rho + 2\eta^{\mu\sigma}\eta^{\nu\tau}k_1^\lambda k_1^\rho - 2\eta^{\mu\nu}\eta^{\sigma\tau}k_1^\lambda k_1^\rho + \\
&2\eta^{\lambda\tau}\eta^{\mu\nu}k_1^\sigma k_1^\rho + 2\eta^{\lambda\sigma}\eta^{\mu\nu}k_1^\tau k_1^\rho + \eta^{\mu\tau}\eta^{\nu\sigma}k_2^\lambda k_1^\rho + \eta^{\mu\sigma}\eta^{\nu\tau}k_2^\lambda k_1^\rho + \eta^{\lambda\tau}\eta^{\nu\sigma}k_2^\mu k_1^\rho + \\
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&\eta^{\lambda\rho}\eta^{\nu\tau}k_1^\sigma k_2^\mu + \eta^{\lambda\nu}\eta^{\rho\tau}k_1^\sigma k_2^\mu + \eta^{\lambda\sigma}\eta^{\nu\rho}k_1^\tau k_2^\mu - \eta^{\lambda\rho}\eta^{\nu\sigma}k_1^\tau k_2^\mu + \eta^{\lambda\nu}\eta^{\rho\sigma}k_1^\tau k_2^\mu + \\
&2\eta^{\nu\rho}\eta^{\sigma\tau}k_2^\lambda k_2^\mu + \eta^{\mu\tau}\eta^{\rho\sigma}k_1^\lambda k_2^\nu + \eta^{\mu\sigma}\eta^{\rho\tau}k_1^\lambda k_2^\nu + \eta^{\lambda\tau}\eta^{\mu\rho}k_1^\sigma k_2^\nu - \eta^{\lambda\rho}\eta^{\mu\tau}k_1^\sigma k_2^\nu + \\
&\eta^{\lambda\mu}\eta^{\rho\tau}k_1^\sigma k_2^\nu + \eta^{\lambda\sigma}\eta^{\mu\rho}k_1^\tau k_2^\nu - \eta^{\lambda\rho}\eta^{\mu\sigma}k_1^\tau k_2^\nu + \eta^{\lambda\mu}\eta^{\rho\sigma}k_1^\tau k_2^\nu + 2\eta^{\mu\rho}\eta^{\sigma\tau}k_2^\lambda k_2^\nu + \\
&2\eta^{\lambda\tau}\eta^{\rho\sigma}k_2^\mu k_2^\nu + 2\eta^{\lambda\sigma}\eta^{\rho\tau}k_2^\mu k_2^\nu - 2\eta^{\lambda\rho}\eta^{\sigma\tau}k_2^\mu k_2^\nu + \eta^{\mu\tau}\eta^{\nu\sigma}k_1^\lambda k_2^\rho + \eta^{\mu\sigma}\eta^{\nu\tau}k_1^\lambda k_2^\rho + \\
&\eta^{\lambda\nu}\eta^{\mu\tau}k_1^\sigma k_2^\rho + \eta^{\lambda\mu}\eta^{\nu\tau}k_1^\sigma k_2^\rho + \eta^{\lambda\nu}\eta^{\mu\sigma}k_1^\tau k_2^\rho + \eta^{\lambda\mu}\eta^{\nu\sigma}k_1^\tau k_2^\rho + 2\eta^{\mu\tau}\eta^{\nu\sigma}k_2^\lambda k_2^\rho + \\
&2\eta^{\mu\sigma}\eta^{\nu\tau}k_2^\lambda k_2^\rho - 2\eta^{\mu\nu}\eta^{\sigma\tau}k_2^\lambda k_2^\rho + 2\eta^{\lambda\nu}\eta^{\sigma\tau}k_2^\mu k_2^\rho + 2\eta^{\lambda\mu}\eta^{\sigma\tau}k_2^\nu k_2^\rho + \eta^{\nu\tau}\eta^{\rho\sigma}k_1^\lambda k_3^\mu + \\
&\eta^{\nu\sigma}\eta^{\rho\tau}k_1^\lambda k_3^\mu - \eta^{\nu\rho}\eta^{\sigma\tau}k_1^\lambda k_3^\mu + \eta^{\lambda\tau}\eta^{\nu\rho}k_1^\sigma k_3^\mu + \eta^{\lambda\nu}\eta^{\rho\tau}k_1^\sigma k_3^\mu + \eta^{\lambda\sigma}\eta^{\nu\rho}k_1^\tau k_3^\mu + \\
&\eta^{\lambda\nu}\eta^{\rho\sigma}k_1^\tau k_3^\mu + \eta^{\nu\tau}\eta^{\rho\sigma}k_2^\lambda k_3^\mu + \eta^{\nu\sigma}\eta^{\rho\tau}k_2^\lambda k_3^\mu + \eta^{\lambda\tau}\eta^{\rho\sigma}k_2^\nu k_3^\mu + \eta^{\lambda\sigma}\eta^{\rho\tau}k_2^\nu k_3^\mu + \\
&\eta^{\lambda\tau}\eta^{\nu\sigma}k_2^\rho k_3^\mu + \eta^{\lambda\sigma}\eta^{\nu\tau}k_2^\rho k_3^\mu + \eta^{\mu\tau}\eta^{\rho\sigma}k_1^\lambda k_3^\nu + \eta^{\mu\sigma}\eta^{\rho\tau}k_1^\lambda k_3^\nu - \eta^{\mu\rho}\eta^{\sigma\tau}k_1^\lambda k_3^\nu + \\
&\eta^{\lambda\tau}\eta^{\mu\rho}k_1^\sigma k_3^\nu + \eta^{\lambda\mu}\eta^{\rho\tau}k_1^\sigma k_3^\nu + \eta^{\lambda\sigma}\eta^{\mu\rho}k_1^\tau k_3^\nu + \eta^{\lambda\mu}\eta^{\rho\sigma}k_1^\tau k_3^\nu + \eta^{\mu\tau}\eta^{\rho\sigma}k_2^\lambda k_3^\nu + \\
&\eta^{\mu\sigma}\eta^{\rho\tau}k_2^\lambda k_3^\nu + \eta^{\lambda\tau}\eta^{\rho\sigma}k_2^\mu k_3^\nu + \eta^{\lambda\sigma}\eta^{\rho\tau}k_2^\mu k_3^\nu + \eta^{\lambda\tau}\eta^{\mu\sigma}k_2^\rho k_3^\nu + \eta^{\lambda\sigma}\eta^{\mu\tau}k_2^\rho k_3^\nu + \\
&2\eta^{\lambda\tau}\eta^{\rho\sigma}k_3^\mu k_3^\nu + 2\eta^{\lambda\sigma}\eta^{\rho\tau}k_3^\mu k_3^\nu - 2\eta^{\lambda\rho}\eta^{\sigma\tau}k_3^\mu k_3^\nu + \eta^{\mu\tau}\eta^{\nu\rho}k_1^\lambda k_3^\sigma + \eta^{\mu\rho}\eta^{\nu\tau}k_1^\lambda k_3^\sigma + \\
&\eta^{\lambda\nu}\eta^{\mu\rho}k_1^\tau k_3^\sigma + \eta^{\lambda\mu}\eta^{\nu\rho}k_1^\tau k_3^\sigma + \eta^{\mu\tau}\eta^{\nu\rho}k_2^\lambda k_3^\sigma + \eta^{\mu\rho}\eta^{\nu\tau}k_2^\lambda k_3^\sigma - \eta^{\mu\nu}\eta^{\rho\tau}k_2^\lambda k_3^\sigma + \\
&\eta^{\lambda\tau}\eta^{\nu\rho}k_2^\mu k_3^\sigma + \eta^{\lambda\nu}\eta^{\rho\tau}k_2^\mu k_3^\sigma + \eta^{\lambda\tau}\eta^{\mu\rho}k_2^\nu k_3^\sigma + \eta^{\lambda\mu}\eta^{\rho\tau}k_2^\nu k_3^\sigma - \eta^{\lambda\tau}\eta^{\mu\nu}k_2^\rho k_3^\sigma + \\
&\eta^{\lambda\nu}\eta^{\mu\tau}k_2^\rho k_3^\sigma + \eta^{\lambda\mu}\eta^{\nu\tau}k_2^\rho k_3^\sigma + 2\eta^{\lambda\rho}\eta^{\nu\tau}k_3^\mu k_3^\sigma + 2\eta^{\lambda\rho}\eta^{\mu\tau}k_3^\nu k_3^\sigma + \eta^{\mu\sigma}\eta^{\nu\rho}k_1^\lambda k_3^\tau + \\
&\eta^{\mu\rho}\eta^{\nu\sigma}k_1^\lambda k_3^\tau + \eta^{\lambda\nu}\eta^{\mu\rho}k_1^\sigma k_3^\tau + \eta^{\lambda\mu}\eta^{\nu\rho}k_1^\sigma k_3^\tau + \eta^{\mu\sigma}\eta^{\nu\rho}k_2^\lambda k_3^\tau + \eta^{\mu\rho}\eta^{\nu\sigma}k_2^\lambda k_3^\tau - \\
&\eta^{\mu\nu}\eta^{\rho\sigma}k_2^\lambda k_3^\tau + \eta^{\lambda\sigma}\eta^{\nu\rho}k_2^\mu k_3^\tau + \eta^{\lambda\nu}\eta^{\rho\sigma}k_2^\mu k_3^\tau + \eta^{\lambda\sigma}\eta^{\mu\rho}k_2^\nu k_3^\tau + \eta^{\lambda\mu}\eta^{\rho\sigma}k_2^\nu k_3^\tau - \\
&\eta^{\lambda\sigma}\eta^{\mu\nu}k_2^\rho k_3^\tau + \eta^{\lambda\nu}\eta^{\mu\sigma}k_2^\rho k_3^\tau + \eta^{\lambda\mu}\eta^{\nu\sigma}k_2^\rho k_3^\tau + 2\eta^{\lambda\rho}\eta^{\nu\sigma}k_3^\mu k_3^\tau + 2\eta^{\lambda\rho}\eta^{\mu\sigma}k_3^\nu k_3^\tau -
\end{aligned}
$$

On-shell 3pt vertex: $k_i^2 = 0, \quad \epsilon(k_i).k_i = 0$

3-point gluon vertex

$$\left\langle \frac{\delta S^3}{\delta A_\mu^{-a} \delta A_\sigma^{-b} \delta A_\rho^{+c}} \right\rangle_{\text{on-shell}} \rightarrow -2if^{abc} \left(k_1{}^\sigma \eta^{\mu\rho} - k_2{}^\mu \eta^{\rho\sigma} \right) . \tag{20}$$

3-point graviton vertex

$$\left\langle \frac{\delta S^3}{\delta \varphi_{\mu\nu}^- \delta \varphi_{\sigma\tau}^- \delta \varphi_{\rho\lambda}^+} \right\rangle_{\text{on-shell}} \rightarrow 4 \left(k_1{}^\sigma \eta^{\mu\rho} - k_2{}^\mu \eta^{\rho\sigma} \right) \left(k_1{}^\tau \eta^{\nu\lambda} - k_2{}^\nu \eta^{\lambda\tau} \right)$$

(

Notice that: $-2if^{abc} \rightarrow 2i(k_1^\tau \eta^{\nu\lambda} - k_2^\nu \eta^{\lambda\tau})$ (Double-Copy)

Gauge-gravity relations

Table 1. Factorizable four-dimensional $\mathcal{N} \geq 1$ supergravity theories arising from straightforward double-copy [28].

#	$\mathcal{N}$	Factors	Supergravity
1	8	$\mathcal{N}=4$ SYM $\otimes$ $\mathcal{N}=4$ SYM	pure $\mathcal{N}=8$ SG
2	6	$\mathcal{N}=4$ SYM $\otimes$ $\mathcal{N}=2$ SYM	pure $\mathcal{N}=6$ SG
3	5	$\mathcal{N}=4$ SYM $\otimes$ $\mathcal{N}=1$ SYM	pure $\mathcal{N}=5$ SG
4	4	$\mathcal{N}=4$ SYM $\otimes$ ($\mathcal{N}=0$ YM $+ n_v$ scalars)	$\mathcal{N}=4$ SG, n_v vector multiplets
5	4	$\mathcal{N}=2$ SYM $\otimes$ $\mathcal{N}=2$ SYM	$\mathcal{N}=4$ SG, 2 vector multiplets
6	3	$\mathcal{N}=2$ SYM $\otimes$ $\mathcal{N}=1$ SYM	$\mathcal{N}=3$ SG, 1 vector multiplet
7	2	$\mathcal{N}=2$ SYM $\otimes$ ($\mathcal{N}=0$ YM $+ n_v$ scalars)	$\mathcal{N}=2$ SG, n_v+1 vector multiplets
8	2	$\mathcal{N}=1$ SYM $\otimes$ $\mathcal{N}=1$ SYM	$\mathcal{N}=2$ SG, 1 hypermultiplet
9	1	$\mathcal{N}=1$ SYM $\otimes$ ($\mathcal{N}=0$ YM $+ n_v$ scalars)	$\mathcal{N}=1$ SG, n_v vector and 1 chiral multiplets

Massive gravitino (J.L. D.-C. and B.Larios)

- J. L. Diaz-Cruz, J. R. Ellis, K. A. Olive and Y. Santoso, "On the Feasibility of a Stop NLSP in Gravitino Dark Matter Scenarios," JHEP **0705**, 003 (2007)
- J. L. Diaz-Cruz and B. O. Larios, "Stop Decay with LSP Gravitino in the final state: $\tilde{t}_1 \to \widetilde{G}\, W\, b$," Eur. Phys. J. C **76**, no. 3, 157 (2016)
- J. L. Diaz-Cruz, B. O. Larios and O. Meza-Aldama, "Introduction to the Massive Helicity Formalism with applications to the EWSM," J. Phys. Conf. Ser. **761**, no. 1, 012012 (2016)
- J. L. Diaz-Cruz and B. O. Larios, "Helicity Amplitudes for massive gravitinos in N=1 Supergravity," J. Phys. G **45**, no. 1, 015002 (2018)
- J. L. Diaz-Cruz and B. O. Larios, "2-, 3- and 4-body stop decays" (in preparation)
- J. L. Diaz-Cruz et al "Higgs, LETs and gravitons",

Scattering Amplitudes For All Masses and Spins
Nima Arkani-Hamed (Princeton, Inst. Advanced Study), Tzu-Chen Huang (Caltech), Yu-tin Huang
NCTS-TH-1714
e-Print: arXiv:1709.04891 [hep-th] | PDF

Massive gravitino pheno (J.L. D.-C. and B.Larios)

Gravitino wave-functions with spinors

$$\tilde{\Psi}^{\mu}_{++}(p) = \frac{\langle r|\gamma^{\mu}|q]}{\sqrt{2}[rq]}\left(|r\rangle + \tilde{m}\frac{|q]}{[rq]}\right),$$

$$\tilde{\Psi}^{\mu}_{--}(p) = \frac{\langle q|\gamma^{\mu}|r]}{\sqrt{2}\langle rq\rangle}\left(|r] + \tilde{m}\frac{|q\rangle}{\langle rq\rangle}\right),$$

$$\tilde{\Psi}^{\mu}_{-}(p) = \sqrt{\frac{2}{3}}\left(\frac{r^{\mu}}{\tilde{m}} - \tilde{m}\frac{q^{\mu}}{s_{qr}}\right)\left(|r] + \tilde{m}\frac{|q\rangle}{\langle rq\rangle}\right) + \frac{1}{\sqrt{3}}\frac{\langle q|\gamma^{\mu}|r]}{\sqrt{2}\langle rq\rangle}\left(|r\rangle + \tilde{m}\frac{|q]}{[rq]}\right),$$

$$\tilde{\Psi}^{\mu}_{+}(p) = \sqrt{\frac{2}{3}}\left(\frac{r^{\mu}}{\tilde{m}} - \tilde{m}\frac{q^{\mu}}{s_{qr}}\right)\left(|r\rangle + \tilde{m}\frac{|q]}{[rq]}\right) + \frac{1}{\sqrt{3}}\frac{\langle r|\gamma^{\mu}|q]}{\sqrt{2}[rq]}\left(|r] + \tilde{m}\frac{|q\rangle}{\langle rq\rangle}\right),$$

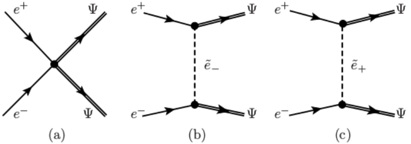


Figure 2: Feynman diagrams for gravitino production at e^+e^- colliders

$$\mathcal{M}^{c}_{-,+} = -\frac{m^2_{\tilde{e}_-}}{F^2}(\mathcal{T}^{t}_{-,+} - \mathcal{T}^{u}_{-,+}) = -\frac{m^2_{\tilde{e}_-}}{F^2}[31]\langle 24\rangle$$

$$\mathcal{M}^{u}_{-,-} = \frac{m^4_{\tilde{e}_-}}{F^2(u - m^2_{\tilde{e}_-})}[41]\langle 23\rangle$$

$$\mathcal{M}^{t}_{-,+} = -\frac{m^4_{\tilde{e}_-}}{F^2(t - m^2_{\tilde{e}_-})}[31]\langle 24\rangle$$

Conclusions

- Great results on Amplitudes shows that out of "Shut up and calculate", we have learned something deep,
- Next? Could we define a QFT by the amplitude methods (good bye to lagrangians? Renormalization?)
- From Massless $\rightarrow$ Massive case? SSB from Amplitudes?,
- Could there be some Math (not QFT) that reproduces Amplitudes?
  ex. Parke-Taylor from WZW model and Twistors ... .

Recollections and reflections

Theme:

- Nature is more beautiful than we think
- Nature is smarter than we are

Mis planes para 2019-20 en MCTP-UNACH - una invitación,

Entre mis propósitos está invitar a los estudiantes y profes a explorar y conocer este maravilloso campo de QFT-Amplitudes.

Ah, pero también pasar de la ciencia al arte y viceversa:

El Rap de la Materia Oscura

(Quiero ser una gran científica)

Autores: J. Lorenzo Díaz Cruz / Karla María Tame Narváez

Personajes principales:

Amanda

Mamá **Papá**

Alberto (Guía) (promotor de la visita a la facultad de ciencias),

Personajes secundarios:

July (compañera de prepa de Amanda)

Carlo (grupo de arte-ciencia)

QFT textbooks - General

- Granpa: Bjorken and Drell, Bogoliubov, ...
- Dad: Itzikson and Zuber, Collins, ...
- Us (young): Veltman notes ($\rightarrow$ "Diagrammatica"), Ramond, ...
- Us (not so young): Peskin, Weinberg (less traditional)
- More recent: Zee, Srednicky, Schwartz, ...

%endcenter

QFT a la Wigner

- Quantum particles states: $|p, \sigma >= U(L(p,k)|k, \sigma >$
- Little group: $Wk = k \rightarrow U(\Lambda)|p, \sigma >= D_{\sigma,\sigma'}(W)|\Lambda p, \sigma' >$,
- Massive particles: $L.G. = SO(3)$,
  Massive particles are labeled by Spin (S) and Mass,
- Massless particles: $L.G. = SO(2) \times T(2)$,
  Massless finite Reprs. are singlets under $T(2) \rightarrow$ Labeled by helicity (e.g. Photon/gluon has $h = \pm 1$, Graviton has $h = \pm 2$,)

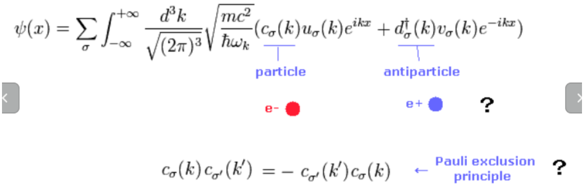
Foundations of On-Shell methods

- The Feynman amplitude is: $M(p_a, \sigma_a) = \delta^4(\Sigma_i p_i) A(p_a, \sigma_a)$

- The amplitude transforms under the Little group:

$$A^{\Lambda}(p_a, \sigma_a) = \Pi_a D_{\sigma_a \sigma_b} A((\Lambda p)_b, \sigma_b) \tag{3}$$

- But fields are manifiestly off-shell and transform as Lorentz tensors, while particles transform under the little group,



Compton effect

$$\mathcal{M}_{+-\lambda_3\lambda_4} = (-i)e^2\langle p_2|\varepsilon^{\mu}_{\lambda_4}(k_4;q_4)(i\gamma_\mu)\left(\frac{-i(\not{p}_1+\not{k}_3)}{(p_1+k_3)^2}\right)(i\gamma_\nu)\varepsilon^{\nu}_{\lambda_3}(k_3;q_3)\,|p_1]$$
$$-(-i)e^2\langle p_2|\varepsilon^{\mu}_{\lambda_3}(k_3;q_3)(i\gamma_\mu)\left(\frac{-i(\not{p}_1+\not{k}_4)}{(p_1+k_4)^2}\right)(i\gamma_\nu)\varepsilon^{\nu}_{\lambda_3}(k_4;q_4)\,|p_1]$$

$$\mathcal{M}_{+-+-} = 2e^2\frac{\langle 2\,4\rangle^2}{\langle 1\,3\rangle\langle 2\,3\rangle}.$$

La amplitud restante $\mathcal{M}_{+--+}$ se encuentra por simetría de cruce $3 \leftrightarrow 4$:

$$\mathcal{M}_{+--+} = 2e^2\frac{\langle 2\,3\rangle^2}{\langle 1\,4\rangle\langle 2\,4\rangle}.$$

El promedio de la amplitud total al cuadrado se calcula de la forma usual

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{4}\left[2\left(|\mathcal{M}_{+-+-}|^2 + |\mathcal{M}_{+--+}|^2\right)\right]$$

Haciendo las cuentas explícitamente

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{4}\left\{2\left[4e^4\left(\frac{\langle 2\,4\rangle^2}{\langle 1\,3\rangle\langle 2\,3\rangle}\right)\left(\frac{\langle 2\,4\rangle^2}{\langle 1\,3\rangle\langle 2\,3\rangle}\right)^* + 4e^4\left(\frac{\langle 2\,3\rangle^2}{\langle 1\,4\rangle 2\,4}\right)\left(\frac{\langle 2\,3\rangle^2}{\langle 1\,4\rangle\langle 2\,4\rangle}\right)^*\right]\right\}$$

$$\langle|\mathcal{M}|^2\rangle = 2e^4\left(\left|\frac{s_{14}}{s_{13}}\right| + \left|\frac{s_{13}}{s_{14}}\right|\right)$$
$$= -2e^4\left(\frac{u}{s} + \frac{s}{u}\right).$$

What is QFT? ("Pert. Theory in Relative space")

- In quantum mechanics one can go from coordinate space to momentum space, i.e. $\Psi(x) \rightarrow \Phi(k)$,
- But QFT seems to make more sense in momentum space,
- Divergenes appear as poles in $\epsilon = D - 4$, and the existence of Quadratic divergences is questionable,
- Dimensional formulation of QFT $\rightarrow$ Space-time must be an emergent phenomena,
- Why take it seriously? Hierarchy problem, Cosmological constant, .. at least,

Free Will in the Theory of Everything[1]

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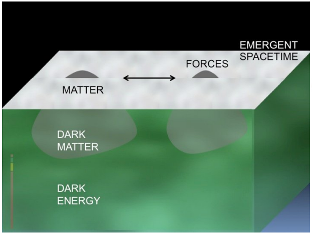
So, being God, you have a second great idea. Before formulating your laws of Nature, you decide about a couple of *demands* that you impose upon your laws of Nature. Tell your computer scientists and mathematicians that they must give you the simplest laws of Nature that comply with your demands.

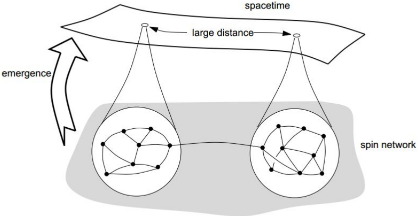
In this paper, it will be argued that very simple demands can be imposed, and that at least some of these demands already lead to a structure that may well resemble our universe. The construction that will eventually emerge will be called "theory of everything". It describes everything that happens in this universe. Now, the first demand that will be suggested here, will appear to be not al all obeyed by the real universe, at first sight. But those are only appearances. Remember that our brains were not designed for this, so wait with your prejudices. We claim to be able to make three observations: one, the set of demands that we will formulate now are nearly inevitable and non negotiable. Two: even though the demands are simple, the mathematical structure of the rules, or laws of physics, emerges to be remarkably complex, possibly too complex for simple humans to grasp. And three: as far as we do understand them, the resulting rules do resemble the laws of Nature governing our real universe. In fact, it may well be that they lead *exactly*

[2]This is only meant metaphorically; this author, fortunately, is not religious.

Emergent phenomena

- Composite models: Higgs, quarks, leptons, gauge bosons... (Bjorken, Harari, Seiberg-Witten..)
- Emergent space-time: modification of general relativity $\rightarrow$ dark matter, dark energy (Verlinde, Nielsen, ..)
- Multiverse: we are an small drop in a vast cosmic ocean .. (String theory)





Foundations of the SM

(a) Gauge principle (Yang-Mills): It provides a rationale for the origin of interactions,

(b) SSB: After the works of Englert-Brout, Higgs, particle physics had a general method to provide masses to gauge vector bosons. From these, it has to be a matter of experiments and model building to find out which model was chosen by nature.

(c) Renormalization of Gauge Theories: 't Hooft and Veltman provided a general method to build renormalizable gauge theories with massive vector bosons.

(d) Anomalies $\rightarrow$ Geometry and QFT.

Non-perurbative QFT → QCD

Many Facets of QCD

- Quantum Field Theory with rich dynamical content
  - ✓ asymptotic freedom, confinement, spontaneous broken chiral symmetry & its restoration at high density, non-trivial vacuum, etc.
- Standard Model of the collective behavior becomes important
  - ✓ phase transition, thermalization, flow, etc.
- Very diverse many-body phenomenology at various limits: